Transforms for High-Rate Distributed Source Coding

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Outline

- Characterize quantizers for distributed source coding at high rates

- Use principles of conventional transform coding in distributed source coding

- Apply new quantization and transformation theory to distributed video coder
Wyner-Ziv Coding

- Rate-distortion theory for distributed source coding suggests small performance loss
  [Slepian, Wolf, 73] [Wyner, Ziv, 76] [Zamir, 96]

- Rate
  \[ R = \frac{1}{n} H(Q|Y) \]

- Distortion
  \[ D = \frac{1}{n} E[||X - \hat{X}||^2] \]
Quantizers for Distributed Coding

- Quantizer $q(x)$ cannot depend on $y$, but statistical dependence between $X$ and $Y$ exploited
- Mapping of different cells into common quantization index may help performance
- In [Rebollo, Zhang, Girod, 03]
  - Rate measure $r(q,y)$ introduced to extend Lloyd algorithm to Slepian-Wolf coding
  - Quantizers found in experiments were uniform
  - Performance close to case in which $Y$ available
State of the Art

- Lossless distributed source coding
  [Pradhan, Ramchandran, 99] [García-Frías, Zhao, 01]
  [Aaron, Girod, 02]

- Quantization for distributed source coding
  - Extension of Lloyd algorithm [Fleming, Zhao, Effros, 01]
  - Further extension for Slepian-Wolf coding [Rebollo, Zhang, Girod, 03]

- Transforms for distributed source coding [Gastpar et al., 03]
  - Conditional covariance matrix constant with side info
  - Not in the context of a practical coding system with quantizers for distributed source coding
High-Rate Quantization - Theorem

Assume traditional high rate results for PDF of $X$ given 
\{ $Y=y$ \}, for each $y$
- Bennett’s assumptions (imply well behaved PDFs)
- Gersho’s conjecture (true if $n=1$)
- Optimal family of lattice quantizers $q(x|y)$ on $x$ for each $y$

Then, there exists asymptotically optimal $q(x)$ for high rate
- Lattice quantizer, no index repetition
- $D \sim M_n \frac{2}{n} h(X|Y) 2^{-2R} (M_n$ normalized moment of inertia, $M_1=1/12$)
- No performance loss by not using $Y$ in quantization
- No performance loss by not using $Y$ in reconstruction (but still used in SW decoder!)
High-Rate Quantization Performance

Source Vector $X$ → $q(x)$ → Slepian-Wolf Encoder → $Q$ → Slepian-Wolf Decoder → $\hat{x}(q)$ → Reconstructed Source Vector $\hat{X}$

Side Information $Y$ → $q(x|y)$ → Cond. Encoder → $Q$ → Cond. Decoder → $\hat{x}(q,y)$ → $\hat{X}$
Transform of Source Data

- Orthonormal transformation
- Rate \( R_i = H(Q_i' | Y) \)  \( R = \frac{1}{n} \sum_i R_i \)
- Distortion \( D_i = E[(X_i' - \hat{X}_i')^2] \)  \( D = \frac{1}{n} \sum D_i \)
- Goal: minimum performance loss w.r.t. joint coding

\[
X' = U^T X \\
\hat{X} = U \hat{X}'
\]
Define $\Sigma_{X|Y} = E_Y[\text{Cov}[X|Y]]$
- Covariance of error of best non-linear estimate $\hat{X}(Y) = E[X|Y]$
- If $E[X|y]$ constant with $y$, then it is just $\text{Cov}[X]$

Assume
- High-rate approximation for each band $i$
- Normalized PDF of transformed components constant with $U$
- Variance of conditional distribution of $X_i$ given $Y$ changes very little with $Y$

Then, optimal rate-distortion performance achieved when
- Uniform quantizer common width in all bands
- $U$ is Karhunen-Loève Transform (KLT) for $\Sigma_{X|Y}$
If $X$ and $Y$ jointly Gaussian
- Only high rate approximation necessary
- Other hypotheses hold exactly, KLT indeed optimal

If $(X_i|\{Y=y\})_i$ wide sense stationary as $n\to\infty$, for each $y$
- Only high rate approximation and PDF invariance necessary
- Discrete Cosine Transform (DCT) asymptotically optimal choice for $U$
Transform of Side Information

\[ Y' = V^T Y \]

- Goal: minimum performance loss by using \( Y_i' \) at each branch instead of \( Y \)
Transform of Side Information - Theorem

- **Assume**
  - $X$ and $Y$ jointly Gaussian
  - High rate approximation

- **Then**
  - Optimal transformation of side info is
    \[
    V^T = U^T \Sigma_{XY} \Sigma_Y^{-1}
    \]
  - Source transformation
  - Estimation of source vector from side info
  - No loss in rate or distortion w.r.t. using entire vector $Y$
Wyner-Ziv DCT Video Coder

For each transform band $i$

Key frames (odd)

$X_i'$

Transform

Scalar Quantizer

Turbo Encoder

Request bits

Buffer

Turbo Decoder

Reconstruction

Inverse Transform

$\hat{X}_i'$

Transform

$Y_i'$

Interpolation/Extrapolation

$Y$

Conventional Intraframe coding

Conventional Intraframe decoding

$K$

$X$

WZ frames (even)
First 100 frames of QCIF Mother and Daughter sequence
- Key frames – odd
- WZ frames – even
- Side information generated from motion-compensated interpolation (MC-I) or extrapolation (MC-E)
- Compared to DCT-based intraframe coding and H.263+ I-B-I-B coding
- Similar step size in all bands
Conclusions

- High-rate quantization for distributed coding
  - Lattice quantizers without index repetition asymptotically optimal
  - Operational Wyner-Ziv rate loss vanishes as $D \to 0$

- Transforms for distributed coding
  - Transformation of the source vector
    - KLT of source vector determined by $\hat{\Sigma}_{X|Y} = E_Y[Cov[X|Y]]$
    - Optimal in the Gaussian case
    - DCT optimal if source process conditionally stationary
  - Transformation of the side information, Gaussian case
    - Transformed estimate of source data given side information
    - No loss in rate or distortion performance

- Experiments show important performance improvement
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Many applications, for instance video coding
High-Rate Quantization

- Gaussian scalar case
- $Y$ noisy version of $X$
- $\text{SNR}_{\text{IN}} = \frac{\sigma_X^2}{\sigma_Z^2} = 5 \text{ dB}$
- $\text{SNR}_{\text{OUT}} = \frac{\sigma_X^2}{D}$

[Rebollo, Zhang, Girod, 03]