Transforms for High-Rate Distributed Source Coding

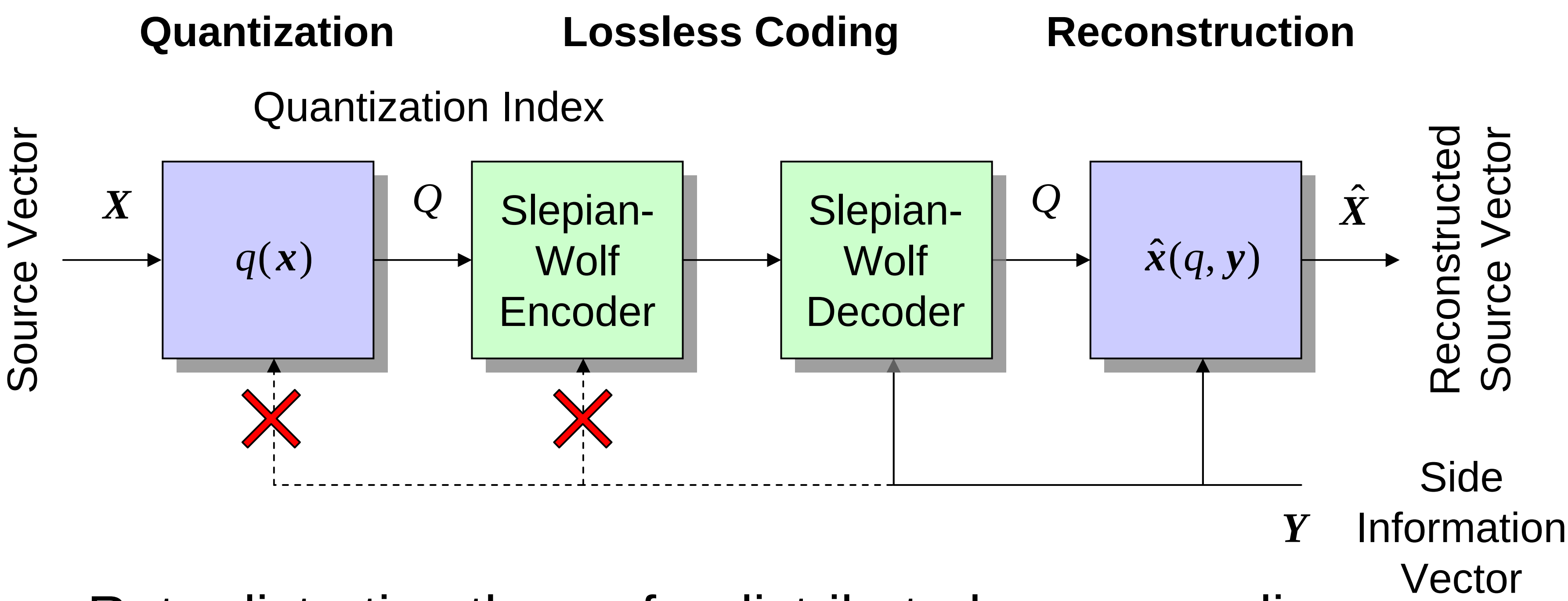
David Rebollo-Monedero, Anne Aaron and Bernd Girod

Information Systems Lab
Dept. of Electrical Eng.
Stanford University

Outline

- Characterize quantizers for distributed source coding at high rates
- Use principles of conventional transform coding in distributed source coding
- Apply new quantization and transformation theory to distributed video coder

Wyner-Ziv Coding



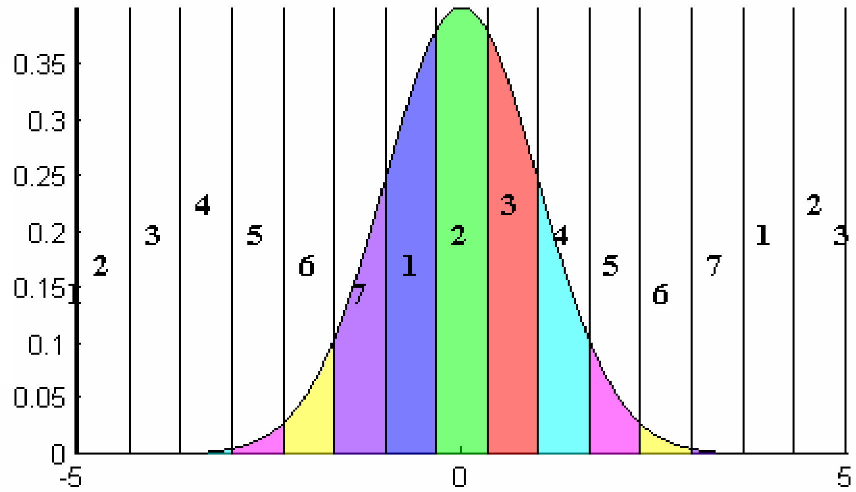
- Rate-distortion theory for distributed source coding suggests small performance loss

  [Slepian, Wolf, 73] [Wyner, Ziv, 76] [Zamir, 96]

- Rate $\mathcal{R} = \frac{1}{n} \mathrm{H}(Q|\boldsymbol{Y})$
- Distortion $\mathcal{D} = \frac{1}{n} \mathrm{E}[\|\boldsymbol{X} - \hat{\boldsymbol{X}}\|^2]$

Quantizers for Distributed Coding

- Quantizater $q(\boldsymbol{x})$ cannot depend on $\boldsymbol{y}$, but statistical dependence between $\boldsymbol{X}$ and $\boldsymbol{Y}$ exploited
- Mapping of different cells into common quantization index may help performance
- In [Rebollo, Zhang, Girod, 03]
  - Rate measure $r(q,\boldsymbol{y})$ introduced to extend Lloyd algorithm to Slepian-Wolf coding
  - Quantizers found in experiments were uniform
  - Performance close to case in which $\boldsymbol{Y}$ available



State of the Art

- Lossless distributed source coding
  [Pradhan, Ramchandran, 99] [García-Frías, Zhao, 01]
  [Aaron, Girod, 02]

- Quantization for distributed source coding
  - Extension of Lloyd algorithm [Fleming, Zhao, Effros, 01]
  - Further extension for Slepian-Wolf coding [Rebollo, Zhang, Girod, 03]

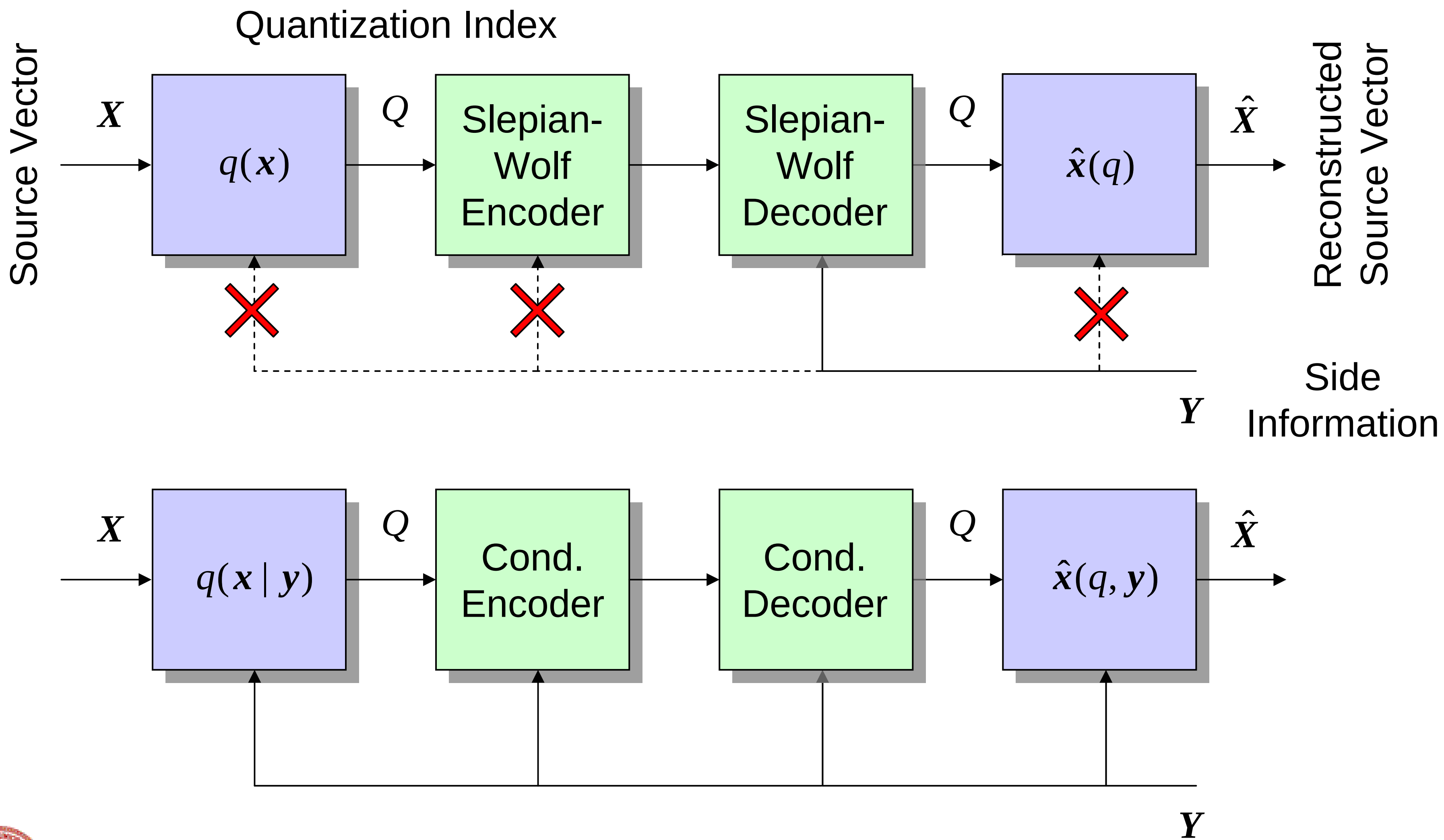
- Transforms for distributed source coding [Gastpar *et al.*, 03]
  - Conditional covariance matrix constant with side info
  - Not in the context of a practical coding system with quantizers for distributed source coding

High-Rate Quantization - Theorem

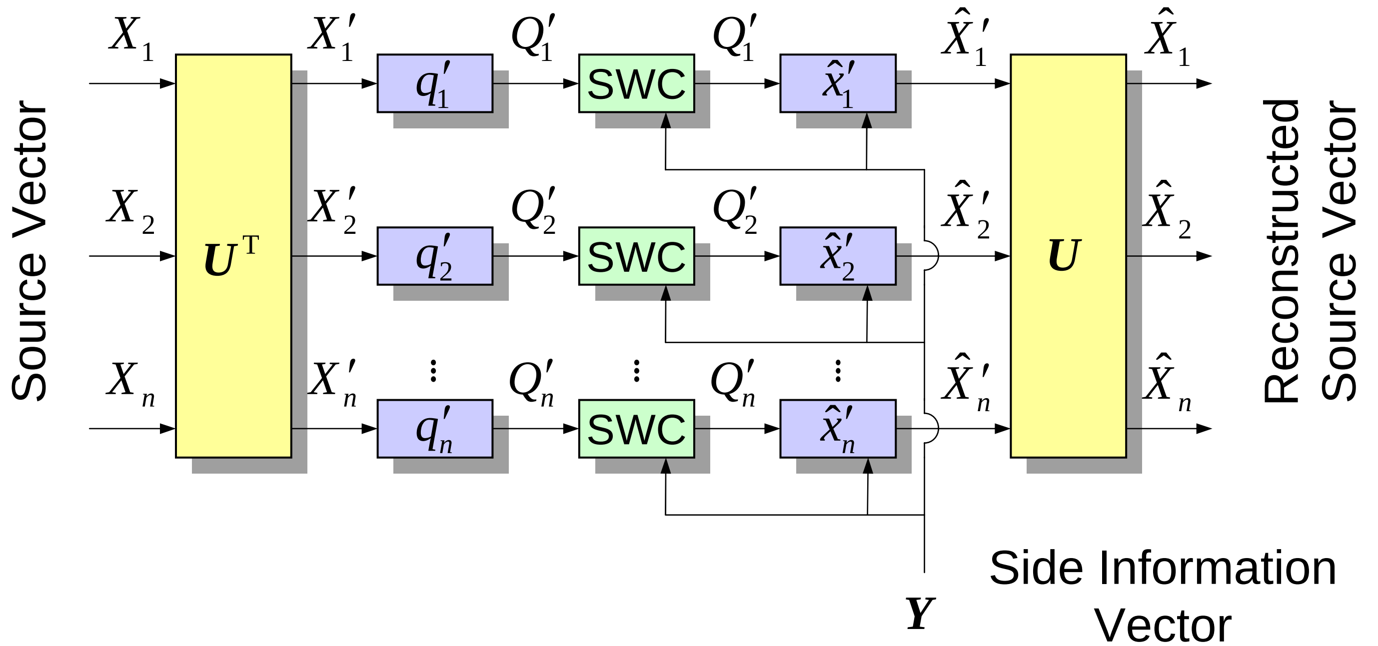
- Assume traditional high rate results for PDF of $\boldsymbol{X}$ given $\{\boldsymbol{Y}=\boldsymbol{y}\}$, for each $\boldsymbol{y}$
  - Bennett's assumptions (imply well behaved PDFs)
  - Gersho's conjecture (true if n=1)
  - Optimal family of lattice quantizers $q(\boldsymbol{x}|\boldsymbol{y})$ on $\boldsymbol{x}$ for each $\boldsymbol{y}$

- Then, there exists asymptotically optimal $q(\boldsymbol{x})$ for high rate
  - Lattice quantizer, no index repetition
  - $\mathcal{D} \simeq M_n \, 2^{\frac{2}{n}\mathrm{h}(\boldsymbol{X}|\boldsymbol{Y})} \, 2^{-2\mathcal{R}}$ (M_n normalized moment of inertia, $M_1=1/12$)
  - No performance loss by not using $\boldsymbol{Y}$ in quantization
  - No performance loss by not using $\boldsymbol{Y}$ in reconstruction (but still used in SW decoder!)

High-Rate Quantization Performance



Transform of Source Data



$$X' = U^T X$$

$$\hat{X} = U \hat{X}'$$

- Orthonormal transformation
- Rate $\mathcal{R}_i = \mathrm{H}(Q_i' | \boldsymbol{Y})$ $\quad \mathcal{R} = \frac{1}{n} \sum_i \mathcal{R}_i$
- Distortion $\mathcal{D}_i = \mathrm{E}[(X_i' - \hat{X}_i')^2]$ $\quad \mathcal{D} = \frac{1}{n} \sum \mathcal{D}_i$
- Goal: minimum performance loss w.r.t. joint coding

Transform of Source Data - Theorem

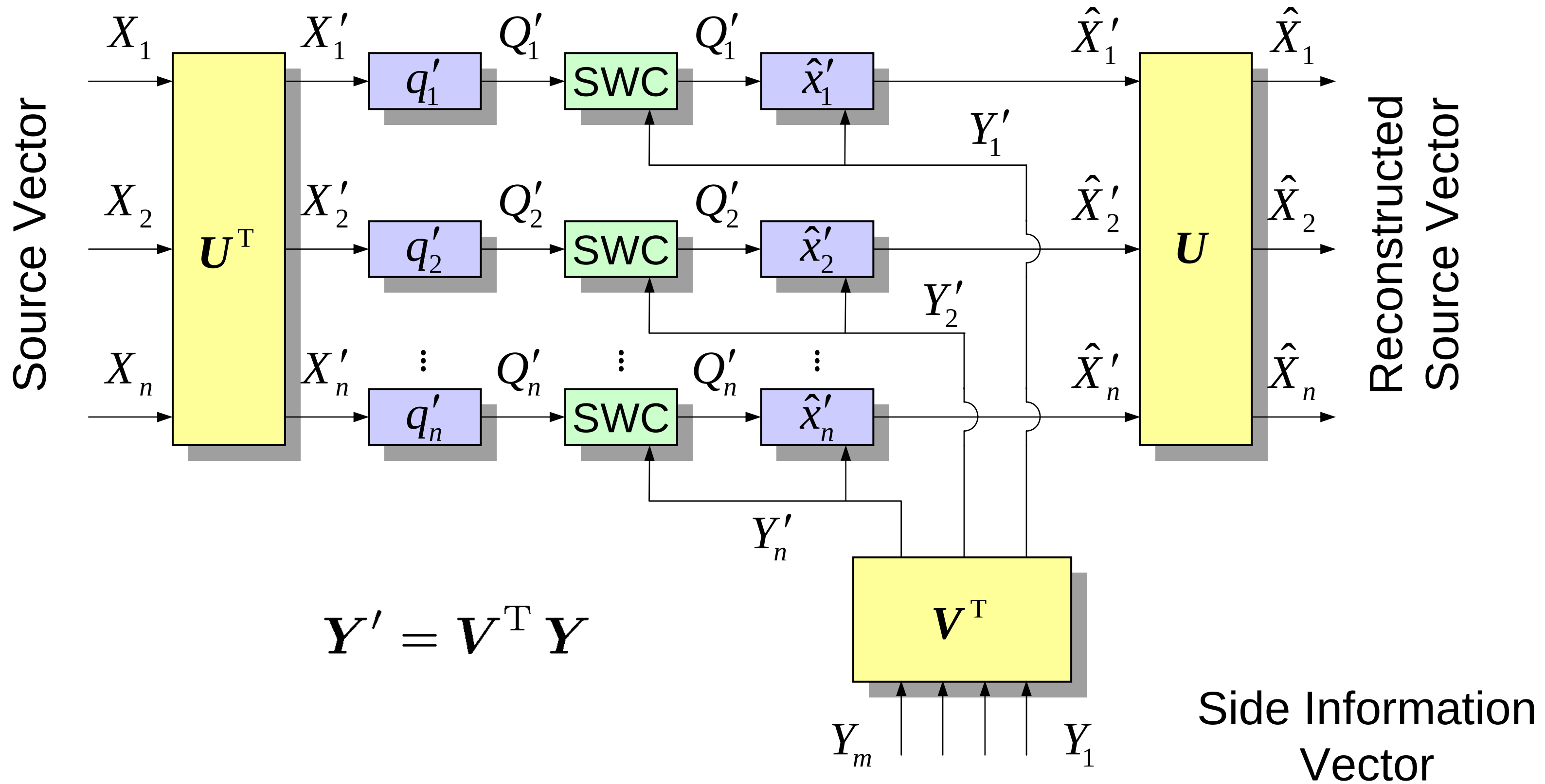
- Define $\bar{\boldsymbol{\Sigma}}_{\boldsymbol{X}|\boldsymbol{Y}} = \mathrm{E}_{\boldsymbol{Y}}[\mathrm{Cov}[\boldsymbol{X}|\boldsymbol{Y}]]$
  - Covariance of error of best non-linear estimate $\hat{\boldsymbol{X}}(\boldsymbol{Y}) = \mathrm{E}[\boldsymbol{X}|\boldsymbol{Y}]$
  - If $\mathrm{E}[\boldsymbol{X}|\boldsymbol{y}]$ constant with $\boldsymbol{y}$, then it is just $\mathrm{Cov}[\boldsymbol{X}]$
- Assume
  - High-rate approximation for each band i
  - Normalized PDF of transformed components constant with $\boldsymbol{U}$
  - Variance of conditional distribution of X_i' given $\boldsymbol{Y}$ changes very little with $\boldsymbol{Y}$
- Then, optimal rate-distortion performance achieved when
  - Uniform quantizer common width in all bands
  - $\boldsymbol{U}$ is Karhunen-Loève Transform (KLT) for $\bar{\boldsymbol{\Sigma}}_{\boldsymbol{X}|\boldsymbol{Y}}$

Transform of Source Data - Corollary

- If $\boldsymbol{X}$ and $\boldsymbol{Y}$ jointly Gaussian
  - Only high rate approximation necessary
  - Other hypotheses hold exactly, KLT indeed optimal

- If $(X_i|\{\boldsymbol{Y}=\boldsymbol{y}\})_{\mathrm{i}}$ wide sense stationary as $n \to \infty$, for each $\boldsymbol{y}$
  - Only high rate approximation and PDF invariance necessary
  - Discrete Cosine Transform (DCT) asymptotically optimal choice for $\boldsymbol{U}$

Transform of Side Information



$$\boldsymbol{Y}' = \boldsymbol{V}^{\mathrm{T}}\boldsymbol{Y}$$

- Goal: minimum performance loss by using Y_i' at each branch instead of $\boldsymbol{Y}$

Transform of Side Information - Theorem

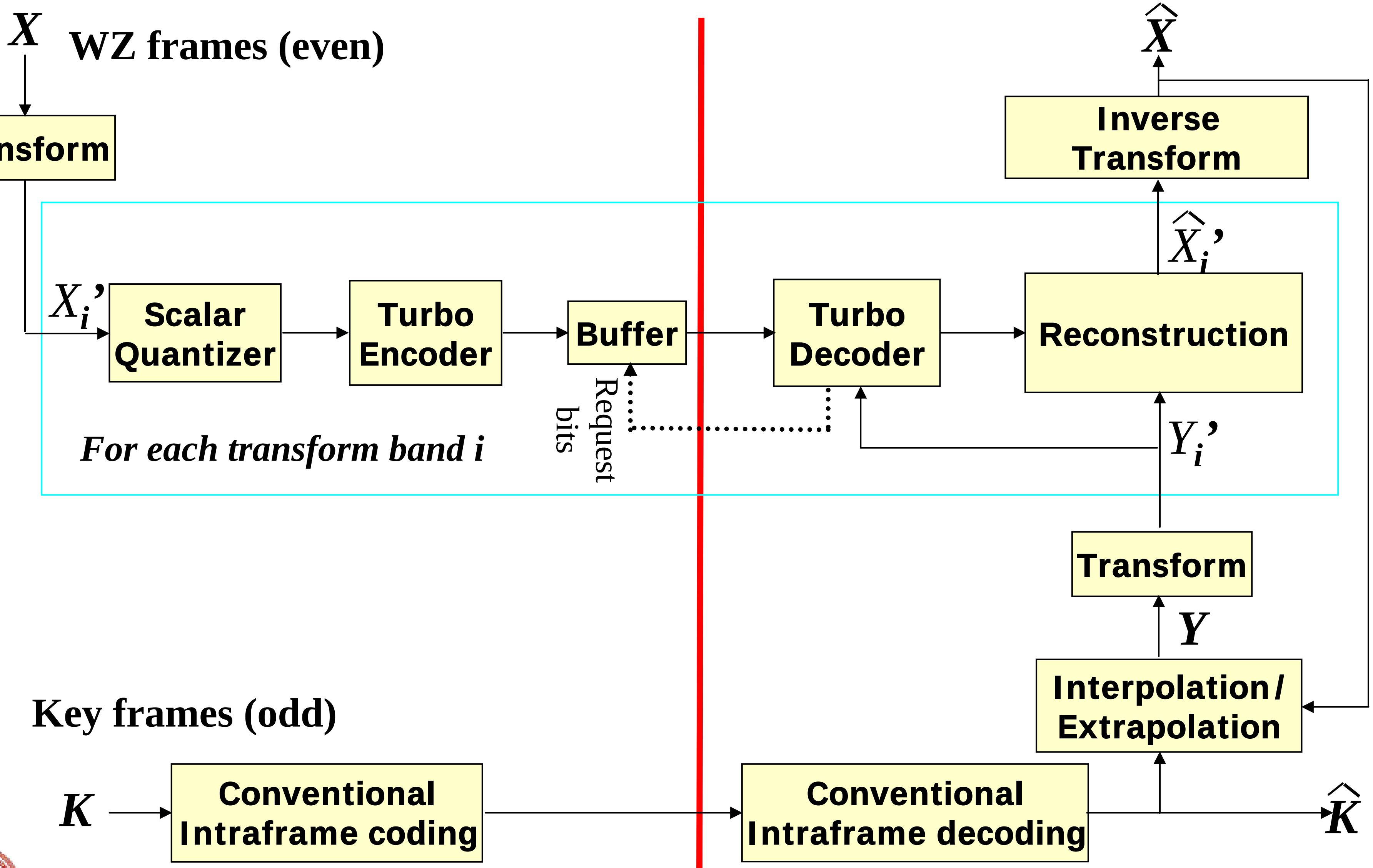
- Assume
  - $\boldsymbol{X}$ and $\boldsymbol{Y}$ jointly Gaussian
  - High rate approximation

- Then
  - Optimal transformation of side info is

$$\boldsymbol{V}^{\mathrm{T}} = \underbrace{\boldsymbol{U}^{\mathrm{T}}}_{\text{Source transformation}} \underbrace{\boldsymbol{\Sigma}_{\boldsymbol{XY}} \boldsymbol{\Sigma}_{\boldsymbol{Y}}^{-1}}_{\text{Estimation of source vector from side info}}$$

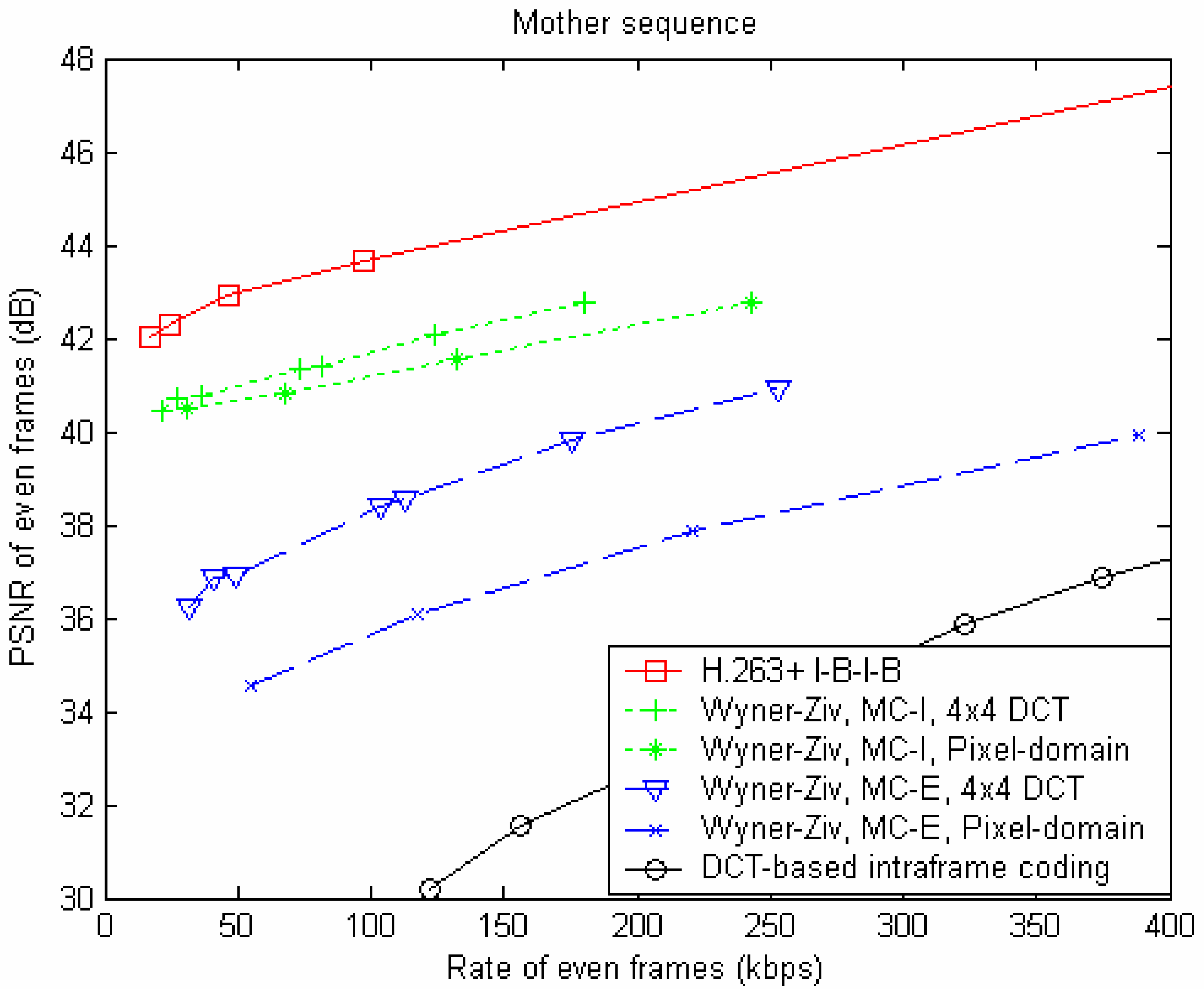
  - No loss in rate or distortion w.r.t. using entire vector $\boldsymbol{Y}$

Wyner-Ziv DCT Video Coder



Mother Sequence: Pixel vs DCT

- First 100 frames of QCIF *Mother and Daughter* sequence
- Key frames – odd
- WZ frames – even
- Side information generated from motion-compensated interpolation (MC-I) or extrapolation (MC-E)
- Compared to DCT-based intraframe coding and H.263+ I-B-I-B coding
- Similar step size in all bands



Conclusions

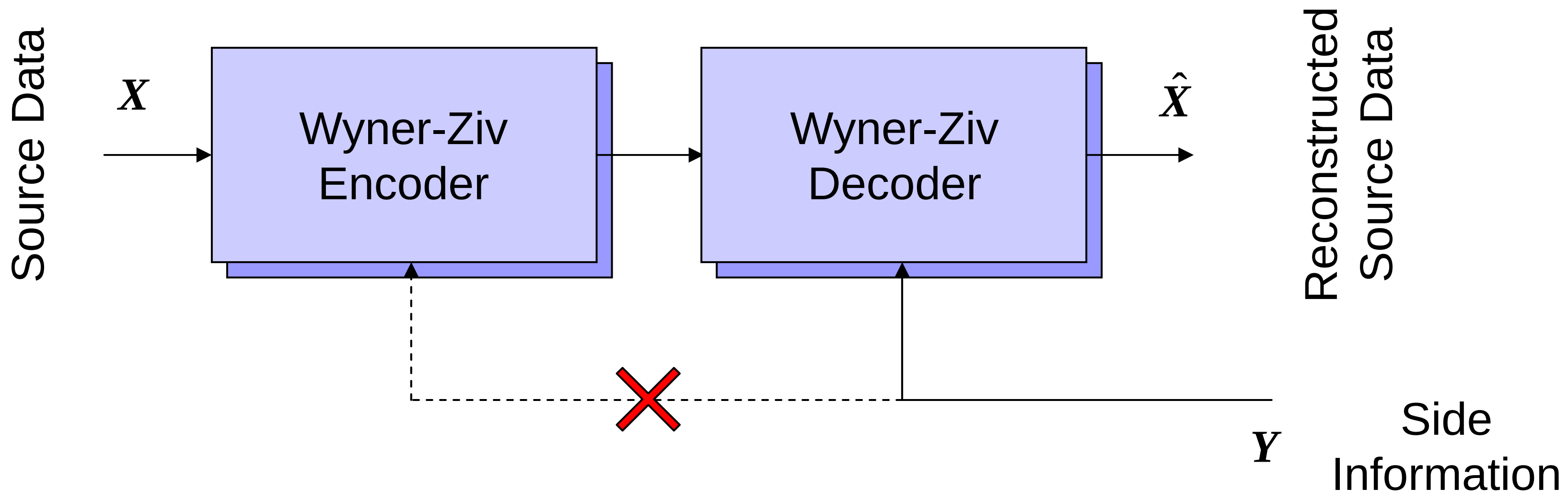
- High-rate quantization for distributed coding
  - Lattice quantizers without index repetition asymptotically optimal
  - Operational Wyner-Ziv rate loss vanishes as $D \to 0$
- Transforms for distributed coding
  - Transformation of the source vector
    - KLT of source vector determined by $\bar{\Sigma}_{\boldsymbol{X}|\boldsymbol{Y}} = \mathrm{E}_{\boldsymbol{Y}}[\mathrm{Cov}[\boldsymbol{X}|\boldsymbol{Y}]]$
    - Optimal in the Gaussian case
    - DCT optimal if source process conditionally stationary
  - Transformation of the side information, Gaussian case
    - Transformed estimate of source data given side information
    - No loss in rate or distortion performance
- Experiments show important performance improvement

Transforms for High-Rate Distributed Source Coding

David Rebollo-Monedero, Anne Aaron and Bernd Girod

Information Systems Lab
Dept. of Electrical Eng.
Stanford University

Wyner-Ziv Coding



- Rate-distortion theory for distributed source coding suggests small performance loss
  [Slepian, Wolf, 73] [Wyner, Ziv, 76] [Zamir, 96]
- Many applications, for instance video coding

High-Rate Quantization

- Gaussian scalar case
- Y noisy version of X
- $\mathrm{SNR}_{\mathrm{IN}} = \frac{\sigma_X^2}{\sigma_Z^2} = 5\,\mathrm{dB}$
- $\mathrm{SNR}_{\mathrm{OUT}} = \frac{\sigma_X^2}{D}$

[Rebollo, Zhang, Girod, 03]